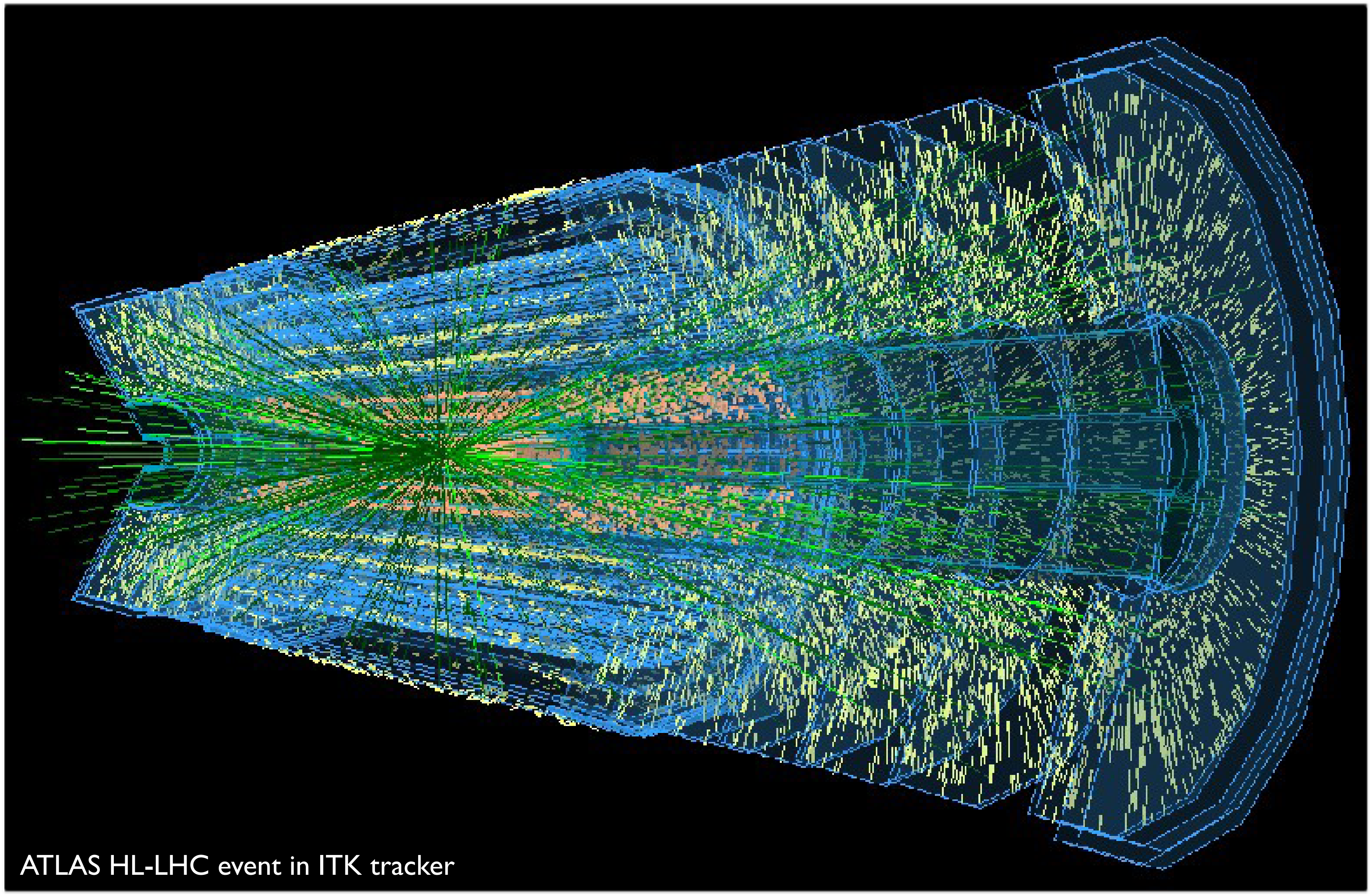
Markus Elsing

Tracking at the LHC (Part 4)

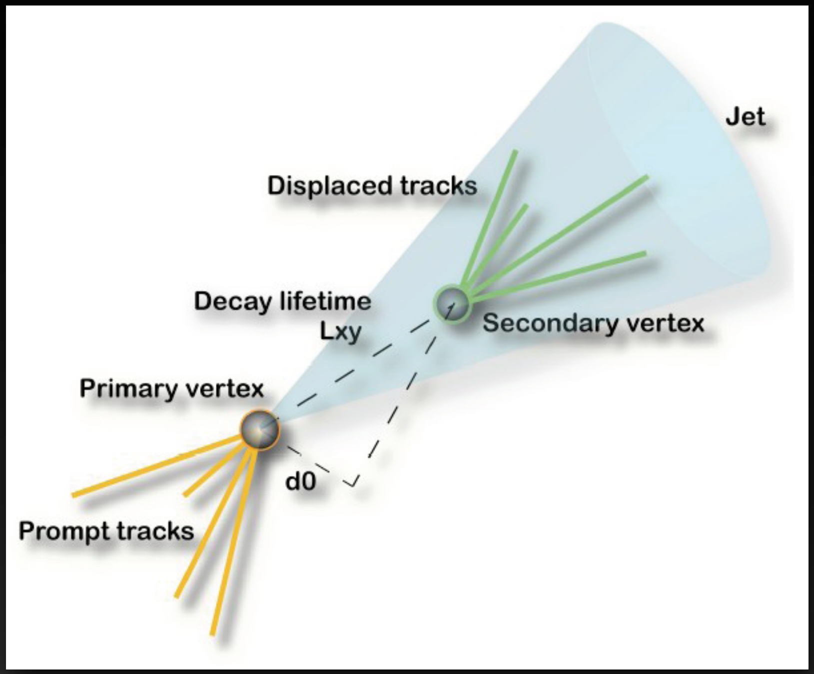
Vertex Reconstruction and its Applications

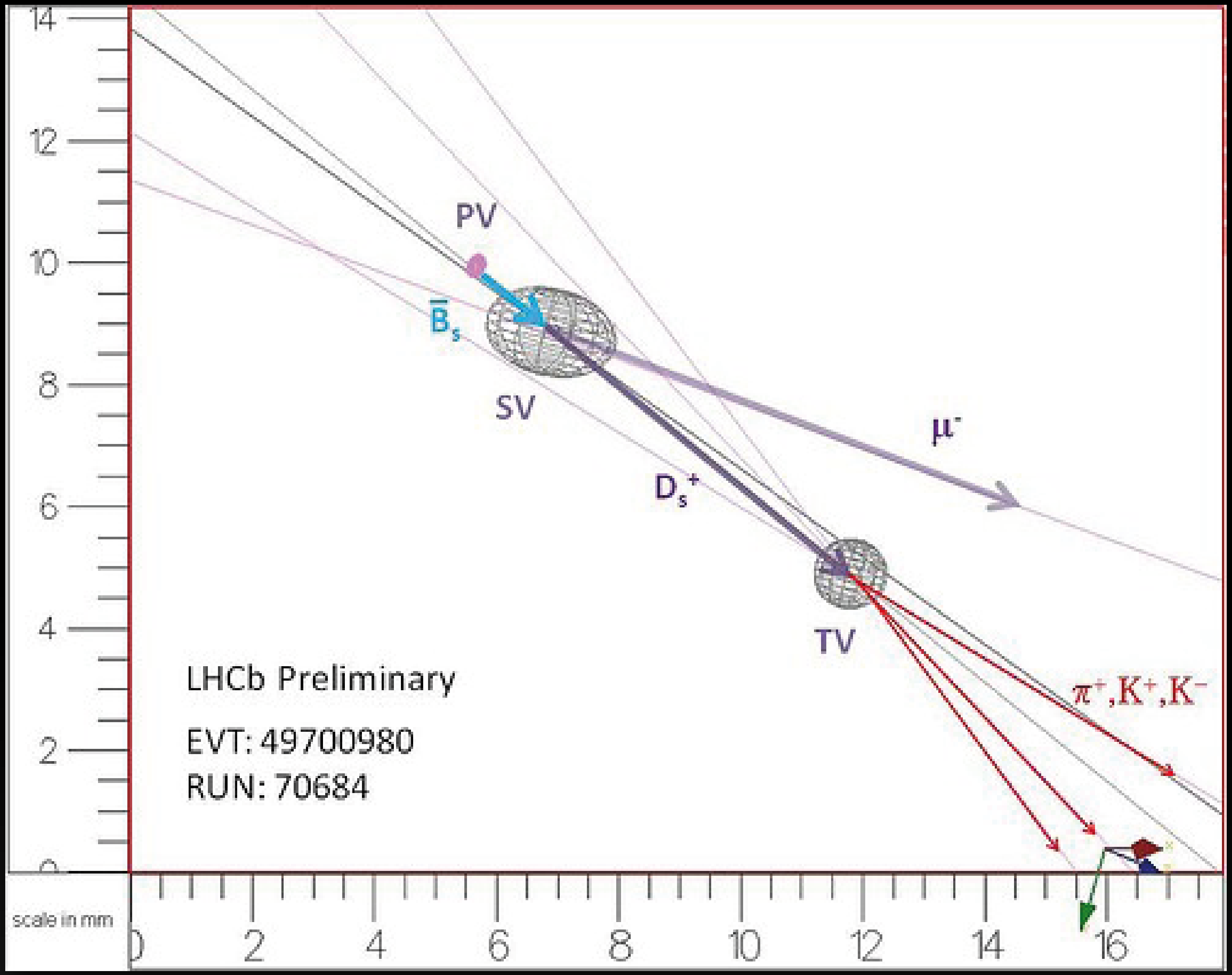


ATLAS HL-LHC event in ITK tracker

Introduction: Vertexing

- b- and c-hadron lifetime
  - ➡ ≈1-1.5 psec (B) and ≈0.4-1psec (D)
  - ➡ tracks have significant impact parameter, **d0** and **z0**
  - ➡ might form a reconstructed secondary vertex

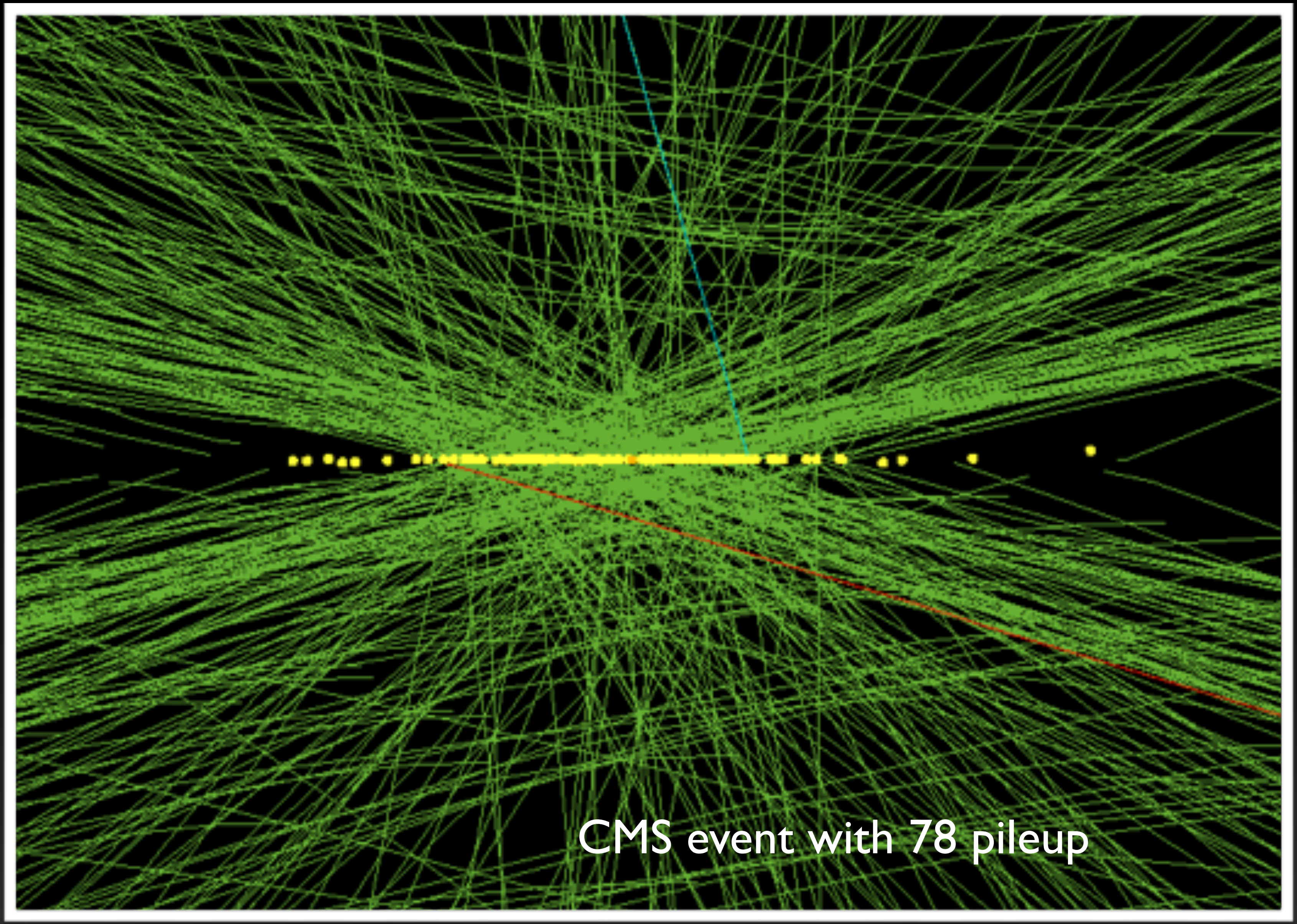




- Example:
  - ➡ a fully reconstructed $B_s \rightarrow D_s \mu\nu \rightarrow KK\pi\mu\nu$ event from LHCb
  - ➡ primary, secondary and tertiary vertex

Event Pileup

- not to forget minimum bias event pileup
  - ➡ nuisance that needs to be managed
  - ➡ affects not only tracking, but as well jet+MET reconstruction, b-tagging, ...



Outline of Part 4

- discuss vertex fitting technique
  - ➡ Least Square and Kalman Filter vertex fitter
  - ➡ adaptive vertex fitting, vertex finding, ZVTOP
- examples for vertexing applications
  - ➡ primary vertexing
  - ➡ Jet-Vertex-Fraction
  - ➡ b-tagging techniques

Vertex Fitting

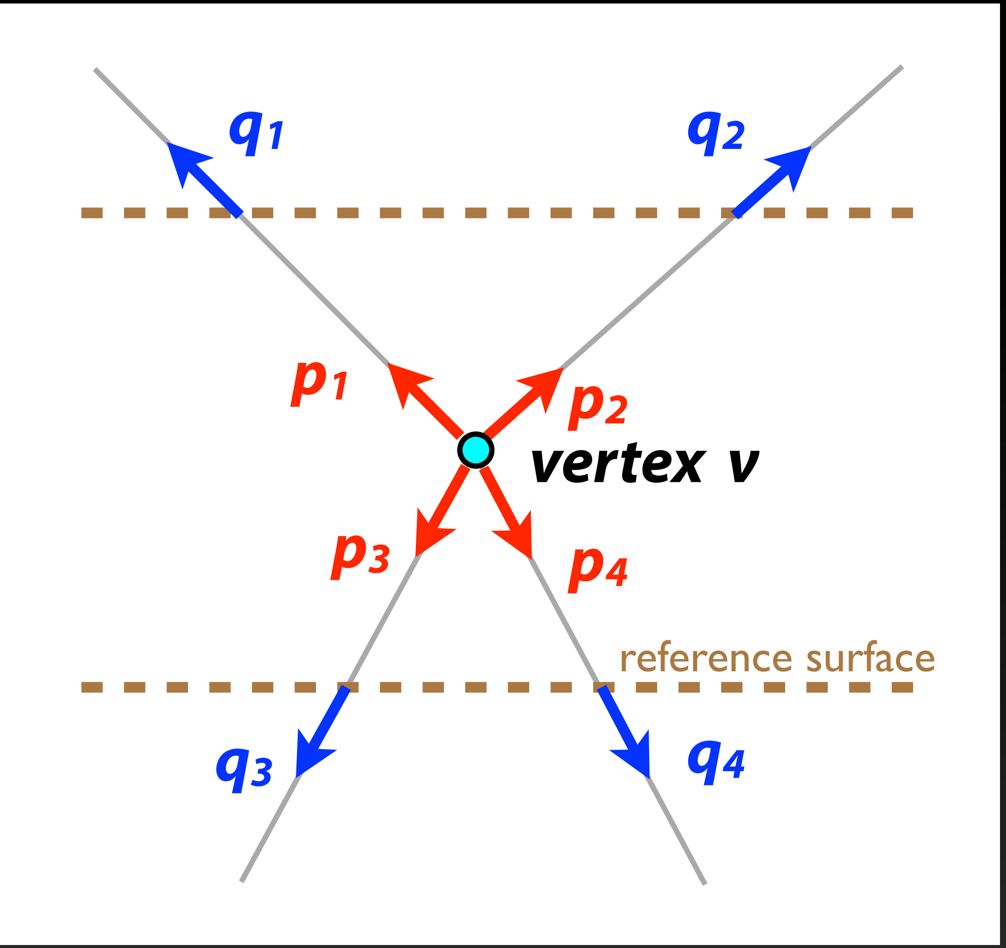
- task of a vertex fit:
  - ➡ estimate the vertex position ***v*** (and the parameters p_k at the vertex) from a set of measured track parameters q_k
- measurement model (similar to track fit)
  - ➡ in mathematical terms:

$$q_i = h_i(v, p_i) + \varepsilon_i$$

with: h_i ~ dependency of track parameters on vertex and parameters at vertex

ε_i ~ error of q_i (noise term)

Jacobians: $A_i = \frac{\partial h_i(v, p_i)}{\partial v}$ $B_i = \frac{\partial h_i(v, p_i)}{\partial p_i}$



  - ➡ in practice: h_i is derived from trajectory model and propagator f:

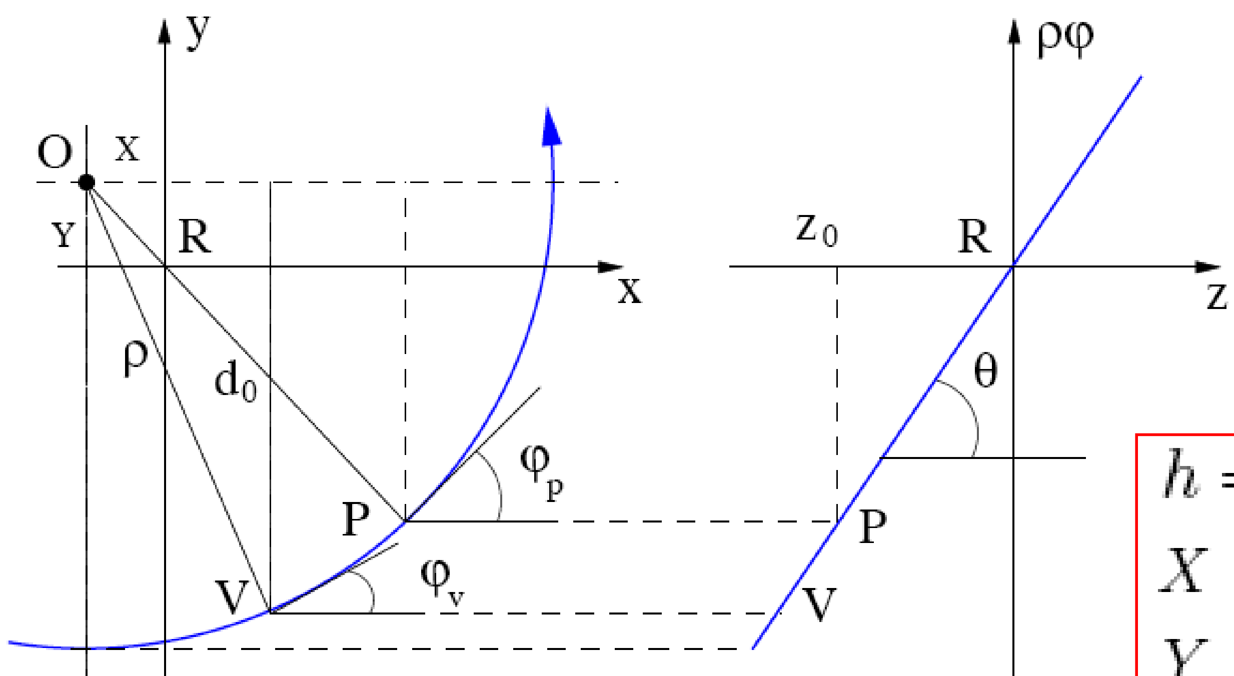
$$h_i = f \circ \tilde{q}(v, p_i) \quad \text{with:} \quad v = (v_x, v_y, v_z), \quad p_i = (\theta_i, \phi_i, Q_i/P_i)$$

  - ➡ commonly used is perigee representation for h_i

G.Piacquadio

Track linearization

- The Kalman filter needs as input the Jacobians and the track parameter initial values which provide a description of the single track linearized around a point.
- We computed the jacobian of the track parameters respect to position and momentum of the track at the vertex in the point of the expansion for ATLAS parameterization for the first time (calculation done with Kirill Prokofiev).



$$
\begin{aligned}
x(\phi) &= x_R - d_{0P}\sin\phi_P + \rho(\sin\phi_P - \sin\phi),\\
y(\phi) &= y_R + d_{0P}\cos\phi_P + \rho(\cos\phi - \cos\phi_P),\\
z(\phi) &= z_R + z_P + \rho\frac{(\phi_P - \phi)}{\tan\theta_P},
\end{aligned}
$$

$$
\begin{aligned}
h &= sign(\rho) & R &= X\cos\phi_V + Y\sin\phi_V\\
X &= x_V - x_R + \rho\sin\phi_V & Q &= X\sin\phi_V - Y\cos\phi_V\\
Y &= y_V - y_R - \rho\cos\phi_V & \Delta\phi &= \phi_P - \phi_V.\\
S &= \sqrt{X^2+Y^2}
\end{aligned}
$$

(definitions)

Results for position and momentum jacobian:

$$
A = \frac{\partial(d_{0P}, z_P, \phi_P, \theta_P, q/p)}{\partial(x_V, y_V, z_V)} = \begin{pmatrix} -h\frac{X}{S} & -h\frac{Y}{S} & 0\\ \frac{\rho}{\tan\theta}\frac{Y}{S^2} & -\frac{\rho}{\tan\theta}\frac{X}{S^2} & 1\\ -\frac{Y}{S^2} & \frac{X}{S^2} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}
$$

$$
B = \frac{\partial(d_{0P}, z_P, \phi_P, \theta_P, q/p)}{\partial(\phi_V, \theta, q/p)} =
$$

$$
\begin{pmatrix} -\frac{h\rho}{S}R & \frac{\rho}{\tan\theta}\left[1-\frac{h}{S}Q\right] & -\frac{\rho}{q/p}\left[1-\frac{h}{S}Q\right]\\ \frac{\rho}{\tan\theta}\left[1-\frac{\rho}{S^2}Q\right] & \rho\left[\Delta\phi+\frac{\rho}{S^2\tan^2\theta}R\right] & \frac{\rho}{q/p\tan\theta}\left[\Delta\phi-\frac{\rho}{S^2}R\right]\\ \frac{\rho}{S^2}Q & -\frac{\rho}{S^2\tan\theta}R & \frac{\rho}{S^2 q/p}R\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{pmatrix}
$$

Least Square Vertex Fit

➡ let's look at the math again....

$$\chi^2 = \sum_i \Delta q_i^T G_i \Delta q_i$$ with: $\Delta q_i = q_i - h_i(v, p_i)$

$V_i = G_i^{-1}$ covariance of the measured q_i

linearize the problem: $v \rightarrow v_0 + \delta v$ and $p_i \rightarrow p_{i,0} + \delta p_i$

$$h_i(v, p_i) \cong h_i(v_0, p_{i,0}) + A_i \delta v + B_i \delta p_i$$ + higher terms

this yields:

$$\chi^2 = \sum_i \left(h_i(v_0, p_{i,0}) + A_i \delta v + B_i \delta p_i\right)^T G_i \left(h_i(v_0, p_{i,0}) + A_i \delta v + B_i \delta p_i\right)$$

minimizing the linearized χ^2 gives the following set of equations:

$$\frac{\partial \chi^2}{\partial v} = 0 \quad \Rightarrow \quad \left(\sum_i A_i^T G_i A_i\right) \cdot \delta v + \sum_i A_i^T G_i B_i \cdot \delta p_i = \sum_i A_i^T G_i \cdot \Delta q_{i,0}$$

$$\frac{\partial \chi^2}{\partial p_i} = 0 \quad \Rightarrow \quad B_i^T G_i A_i \cdot \delta v + B_i^T G_i B_i \cdot \delta p_i = B_i^T G_i \cdot \Delta q_{i,0}$$

with: $\Delta q_{i,0} = q_i - h_i(v_0, p_{i,0})$

➡ system of (i+1) linear matrix equations which can be solved

Least Square Vertex Fit

➡ so let's solve the system...

$$\left(\sum_i A_i^T G_i A_i\right)\cdot \delta v + \sum_i A_i^T G_i B_i \cdot \delta p_i = \sum_i A_i^T G_i \cdot \Delta q_{i,0} \qquad (1)$$

$$B_i^T G_i A_i \cdot \delta v + B_i^T G_i B_i \cdot \delta p_i = B_i^T G_i \cdot \Delta q_{i,0} \qquad (2)$$

transform (2) to replace δp_i in equation (1), gives:

$$\delta v = C \cdot \sum_i A_i^T G_i^B \cdot \Delta q_{i,0}$$

with:

$$G_i^B = G_i - G_i B_i^T W_i B_i G_i$$

$$W_i = \left(B_i^T G_i B_i\right)^{-1}$$

and $C = \left(\sum_i A_i^T G_i^B A_i\right)^{-1}$ covariance of v

➡ usually one iterates the fit to ensure convergence

Least Square Vertex Fit

➡ so let's solve the system...

$$\left(\sum_i A_i^T G_i A_i\right)\cdot \delta v + \sum_i A_i^T G_i B_i \cdot \delta p_i = \sum_i A_i^T G_i \cdot \Delta q_{i,0} \quad (1)$$

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➡ usually one iterates the fit to ensure convergence

➡ still have to compute the corrections to p_i

➡ but: can obtain a **faster vertex fit**, if we neglect the δp_i terms

Least Square Vertex Fit

➡ compute the corrections to p_i

$$\left(\sum_i A_i^T G_i A_i\right)\cdot \delta v + \sum_i A_i^T G_i B_i \cdot \delta p_i = \sum_i A_i^T G_i \cdot \Delta q_{i,0} \quad (1)$$

$$B_i^T G_i A_i \cdot \delta v + B_i^T G_i B_i \cdot \delta p_i = B_i^T G_i \cdot \Delta q_{i,0} \quad (2)$$

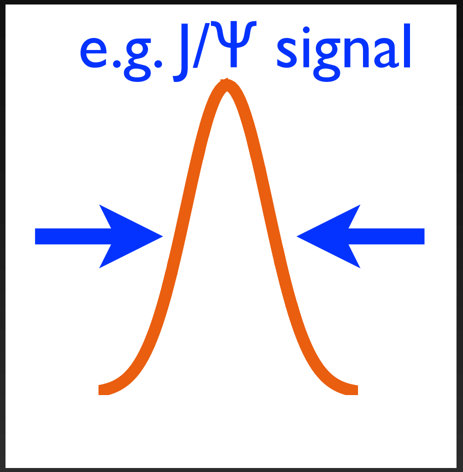
use δv in equation (2) to compute δp_i, gives:

$$\delta p_i = W_i B_i^T G_i \cdot \left(\Delta q_{i,0} - A_i \delta v\right)$$

and $D_i = W_i + W_i B_i^T G_i A_i C A_i^T G_i B_i W_i$ covariance of δp_i

➡ vertex fit is used to improve momentum measurement at vertex

➡ used to improve invariant mass resolution for reconstructed decays



Kalman Filter Notation

➡ the least square vertex fit can as well be written as a progressive fit
➡ results in an extended Kalman Filter vertex fit

1. Let's assume δv_{i-1} has been estimated using i-1 tracks. Track i is added using the update equations:

$$\delta v_i = C_i^{-1} \cdot \left[C_{i-1} \delta v_{i-1} + A_i^T G_i^B \cdot \Delta q_{i,i-1} \right]$$

and the covariance of δv_i is:

$$C_i = \left(C_{i-1}^{-1} + A_i^T G_i^B A_i \right)^{-1}$$

2. update to parameters is:

$$\delta p_{i,i} = W_i B_i^T G_i \cdot \left(\Delta q_{i,i-1} - A_i \delta v_i \right)$$

and the covariance of $\delta p_{i,i}$:

$$D_i = W_i + W_i B_i^T G_i A_i C_i A_i^T G_i B_i W_i$$

Billoir, Fruhwirth, Catlin et al.

weight matrix notation

➡ the smoother in this case is equivalent to computing the parameters $q_{i,n}$ from the final vertex estimate δv_n and $\delta p_{i,n}$

$$q_{i,n} = h_i(v_0 + \delta v_n, p_{i,0} + \delta p_{i,n})$$

with: $\mathrm{cov}(q_{i,n}) = B_i W_i B_i^T + V_i^B G_i A_i C_n A_i^T G_i V_i^B$ and $V_i^B = V_i - B_i W_i B_i^T$

Beam Spot Constraint Fit

- ➡ beam spot ***b*** and its covariance matrix ***E_b^{-1}*** determined externally
- ➡ use information in fit as external constraint
  - straight forward in Kalman Filter vertex fit, its the starting vertex

$$\delta v_0 = b \quad \text{and} \quad C_0 = E_b^{-1}$$

  - in a Least Square vertex fit an additional term is added to the χ^2

$$\chi^2 = \sum_i \Delta q_i^T G_i \Delta q_i + (b - v)^T E_b (b - v)$$

minimizing the linearize χ^2 leads to the modified set of equations:

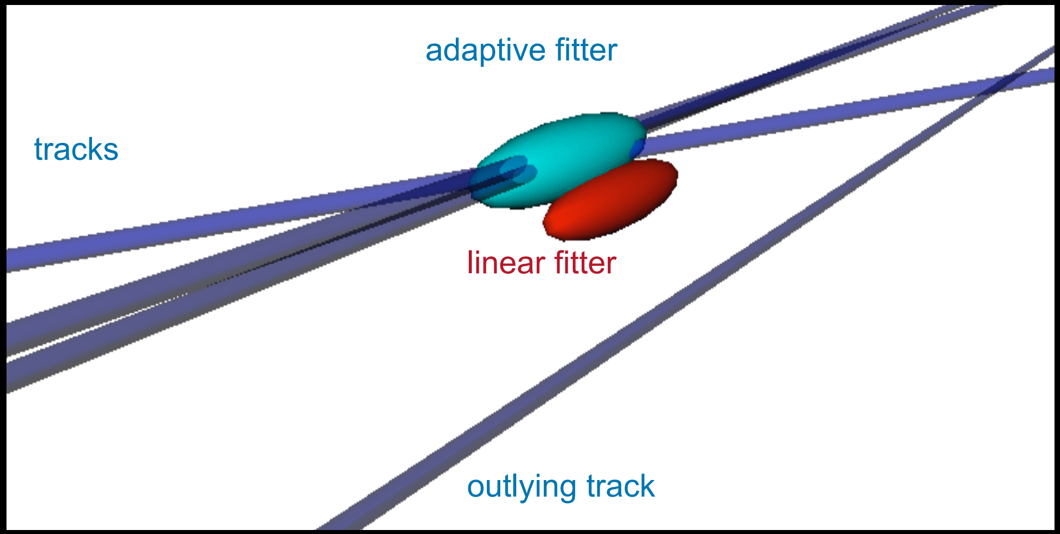
$$\left(E_b + \sum_i A_i^T G_i A_i \right) \cdot \delta v + \sum_i A_i^T G_i B_i \cdot \delta p_i = E_b (b - v_0) + \sum_i A_i^T G_i \cdot \Delta q_{i,0} \qquad (1')$$

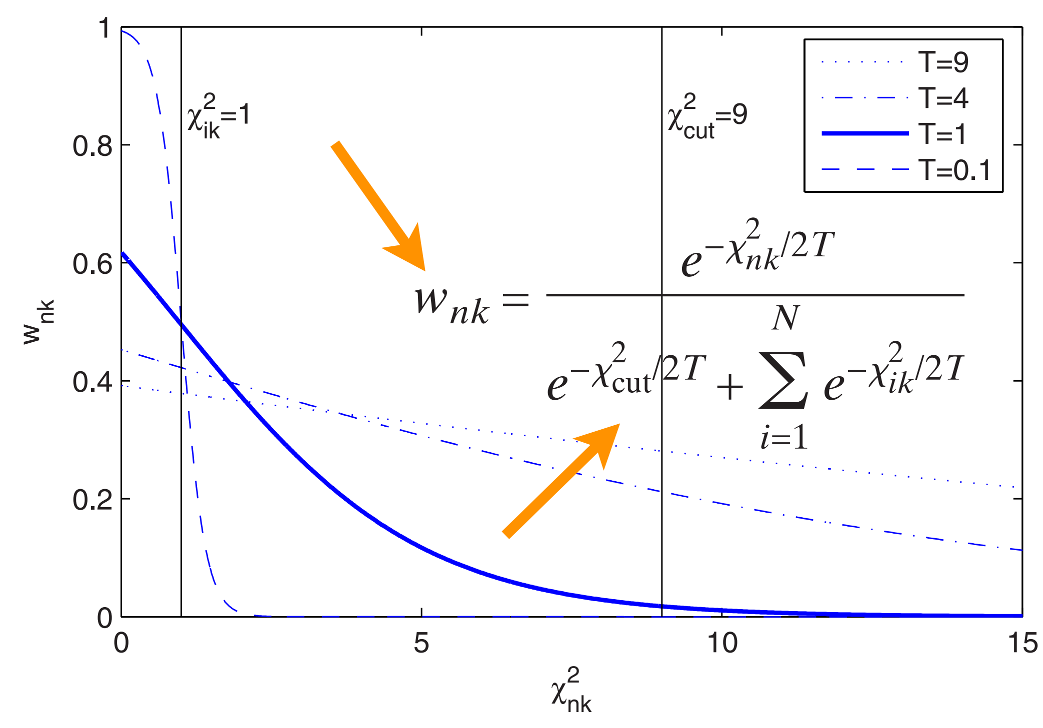
$$B_i^T G_i A_i \cdot \delta v + B_i^T G_i B_i \cdot \delta p_i = B_i^T G_i \cdot \Delta q_{i,0} \qquad (2)$$

which can be solved as before...

Adaptive Vertex Fitter

- adaptive technique
  - ➡ concept used for adaptive track fitting
  - ➡ can be applied as well on vertex fitting
- algorithm is called Adaptive Vertex Fitter
  - ➡ ATLAS and CMS vertexing packages
  - ➡ implemented as iterative, reweighted Kalman filter
    - w_{nk} is weight of track k w.r.t. vertex n
    - outlying tracks are down-weighted automatically
  - ➡ robust fitter !
- extension for Multi-Vertex-Fitter
  - ➡ vertices compete for tracks





Inspecting Outliers

- common problem:
  - ➡ fit quality is bad, want to identify the χ^2 contribution of each track to overall fit (and to track with largest contribution)
  - ➡ compare χ^2 of fit to all tracks with the χ^2 of fit with 1 track less:

$$\Delta\chi_i^2 = \underbrace{\sum_i \Delta q_i^T \cdot G_i \cdot \Delta q_i}_{\text{track } \chi^2} + \underbrace{\left(\Delta q_i - A_i \delta v\right)^T \cdot G_i^B A_i C^{-1} A_i^T G_i^B \cdot \left(\Delta q_i - A_i \delta v\right)}_{\text{change to } \chi^2 \text{ from including it in } \delta v}$$

- application: **iterative vertex finder**
  - ➡ fit all tracks into 1 vertex
  - ➡ remove worst track one by one, until fit χ^2 is acceptable
  - ➡ take removed tracks and try to find next vertex
  - ➡ repeat until no further vertex with at least 2 tracks can be found

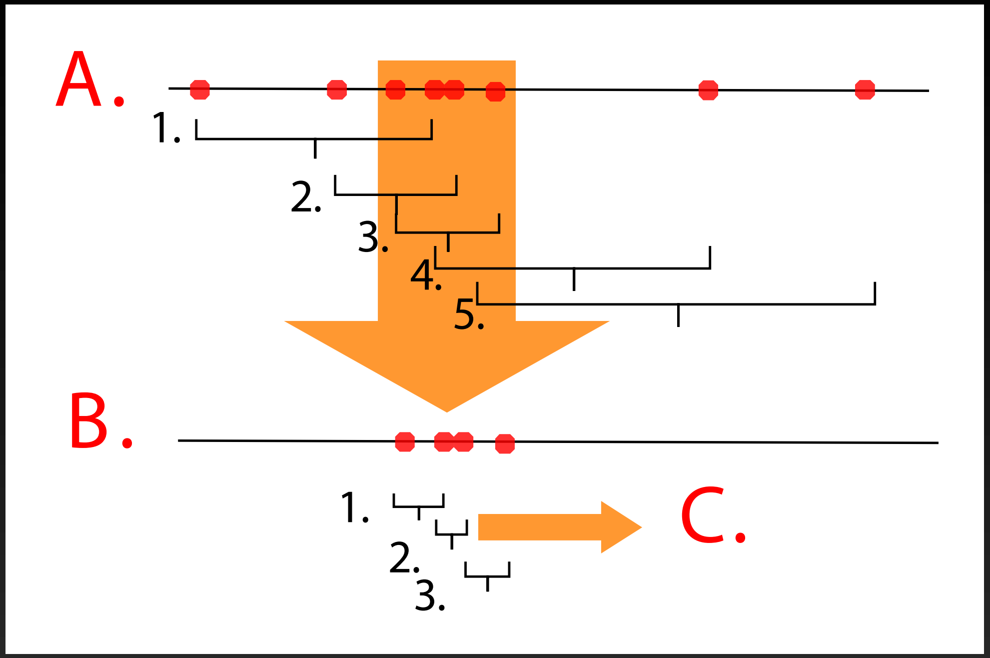
Other Vertex Finding Strategies

- vertex z-scan
  - ➡ used e.g. in primary vertex finding
  - ➡ histogram technique
  - ➡ search for peaks in z_0 of tracks extrapolated to beam line
  - ➡ seed vertex fitter with matching tracks

- half sample mode algorithm
  - ➡ find points of closest approach between all track pairs
  - ➡ in each of the 3 projections:

A. try all the intervals which cover 50 % of the points and take the smallest one (in this case number 3)
B. continue iterating until you have ≤ 3 points left
C. take the mean of the 2 or 3 remaining points

- ➡ defines vertex seed, find matching tracks...



Topological Vertex Finder (ZVTOP)

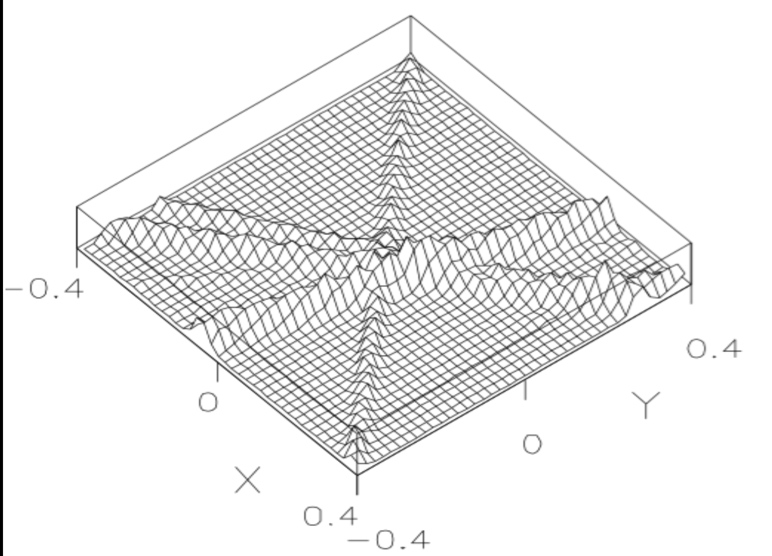
- example for an inclusive vertex finder
  - ➡ very powerful, developed by SLD
- 3 dimensional vertex search
  - ➡ construct for each track Gaussian probability tube ***$f_i(v)$***

$$f_i(\boldsymbol{v}) = \exp\left[-\frac{1}{2}(\boldsymbol{v}-\boldsymbol{r})^{\mathrm{T}} \boldsymbol{V}_i^{-1}(\boldsymbol{v}-\boldsymbol{r}) \right]$$

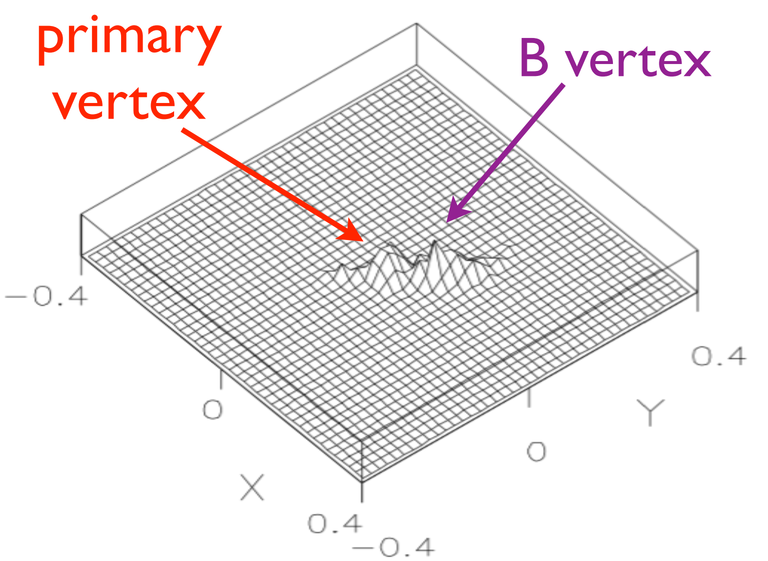
  - ***r*** is point of closest approach of track ***i*** to point ***v***
  - ➡ find all points where ***$f_i(v)$*** is significant for 2 tracks
  - ➡ define vertex probability function ***V(v)*** around those points

$$V(\mathbf{r}) = \sum_{i=0}^{N} f_i(\mathbf{r}) - \frac{\sum_{i=0}^{N} f_i^2(\mathbf{r})}{\sum_{i=0}^{N} f_i(\mathbf{r})}$$

  - search for maxima, merge nearby vertex candidates
  - run vertex fit on them



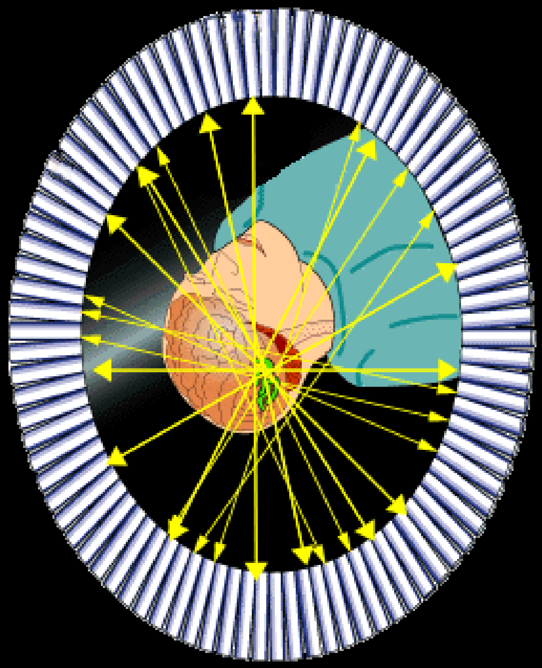
track Gaussian tube function $f_i(v)$



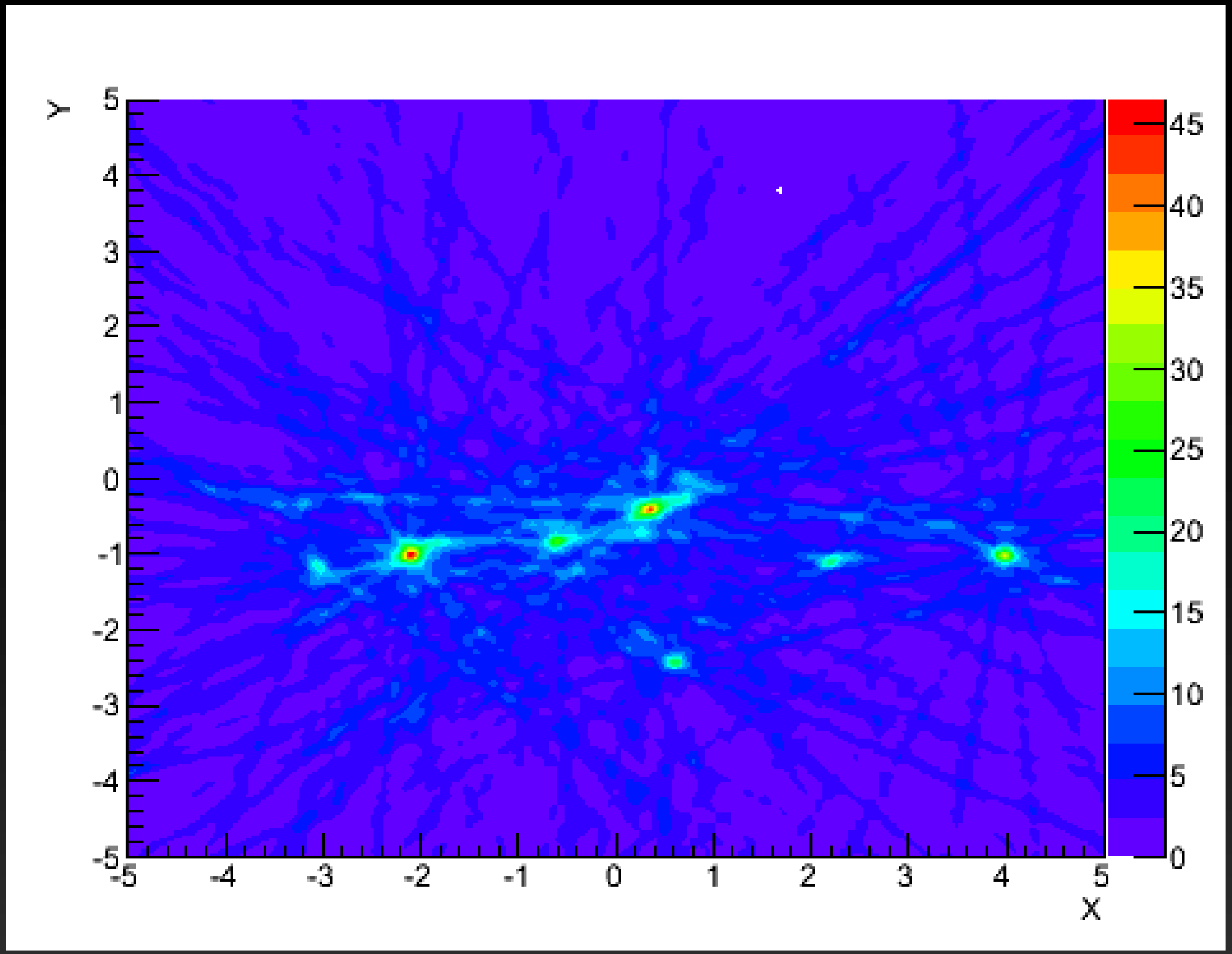
vertex probability function $V(v)$

- SLD used ZVTOP as well to construct an inclusive b-jet tagger

Medical Imaging inspired Vertexing



- based on inverse Radon transformation
  - ➡ used in PET scans
  - ➡ vertex finder is essentially a variant of ZVTOP
    - see: 2012 J. Phys.: Conf. Ser. 396 022021
  - ➡ potentially faster with high pileup
    - evaluated right now e.g. in ATLAS
- steps for vertex finder:
  - ➡ create 3D track density maps



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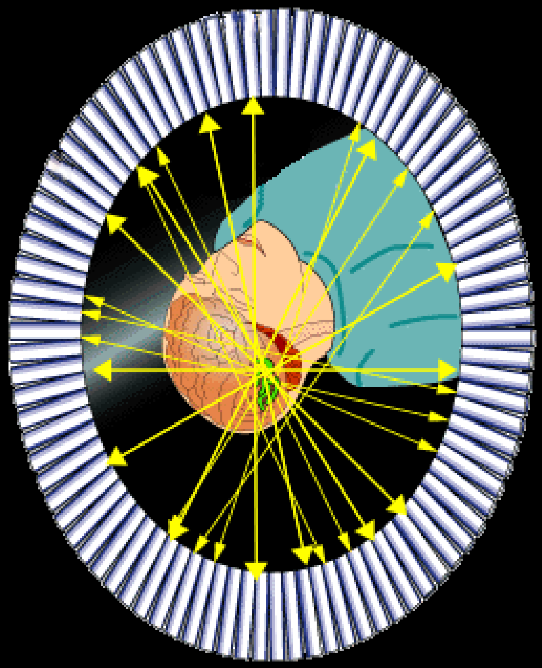

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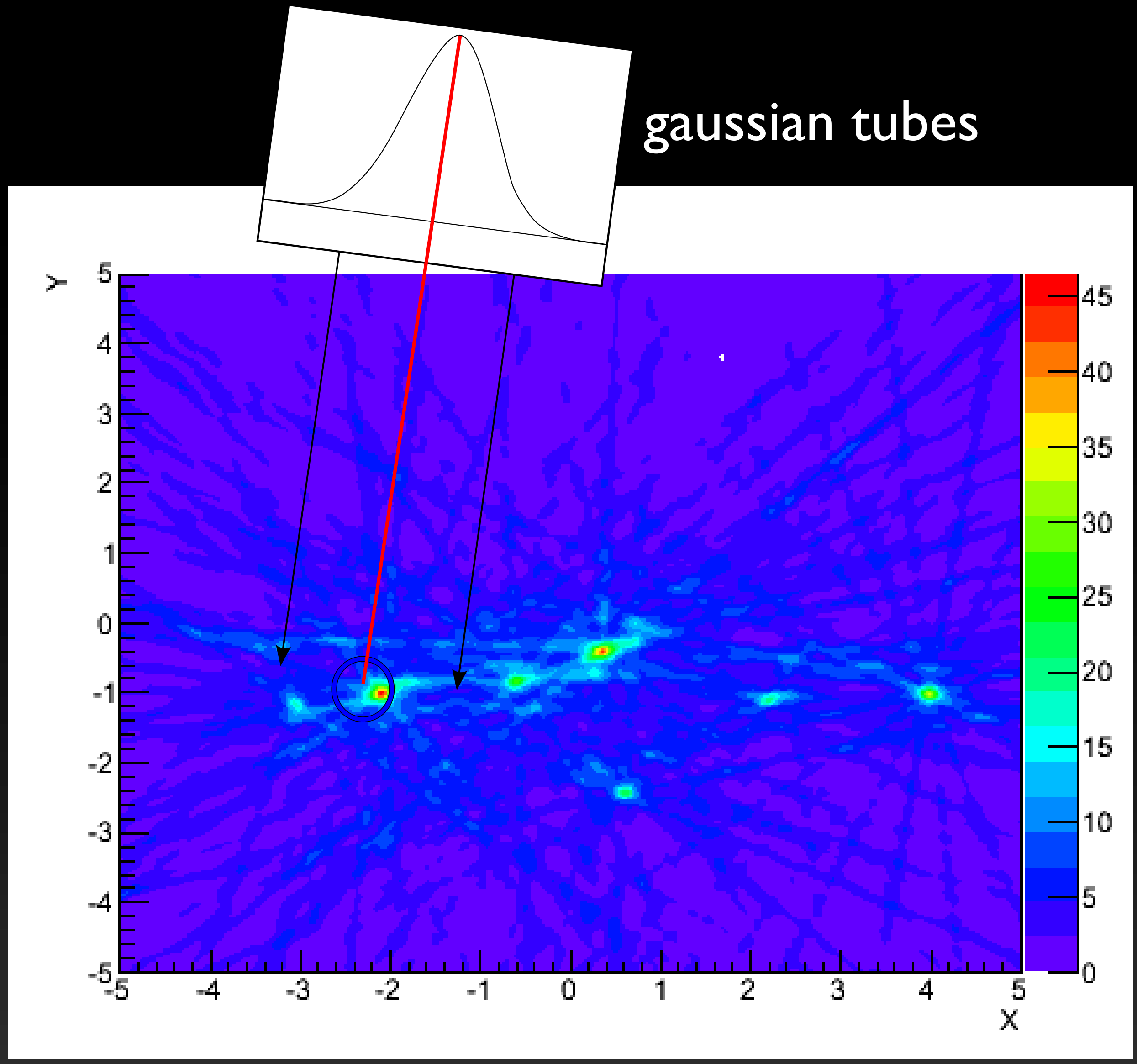
  - ➡ create 3D track density maps
  - ➡ Fourier transform Gaussian tubes

gaussian tubes



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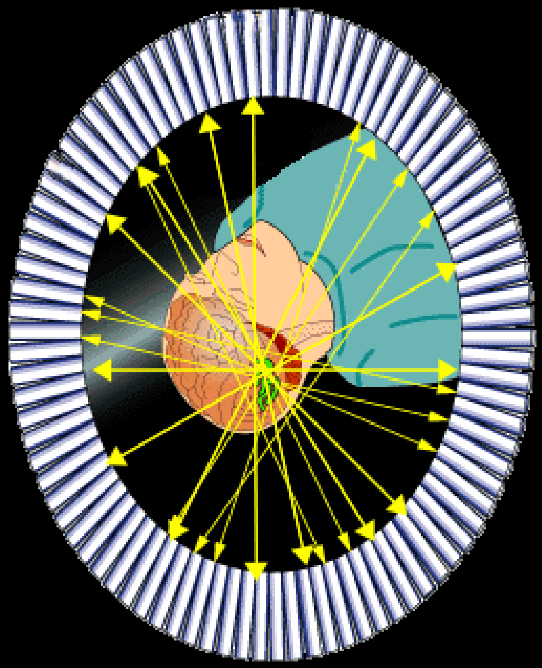

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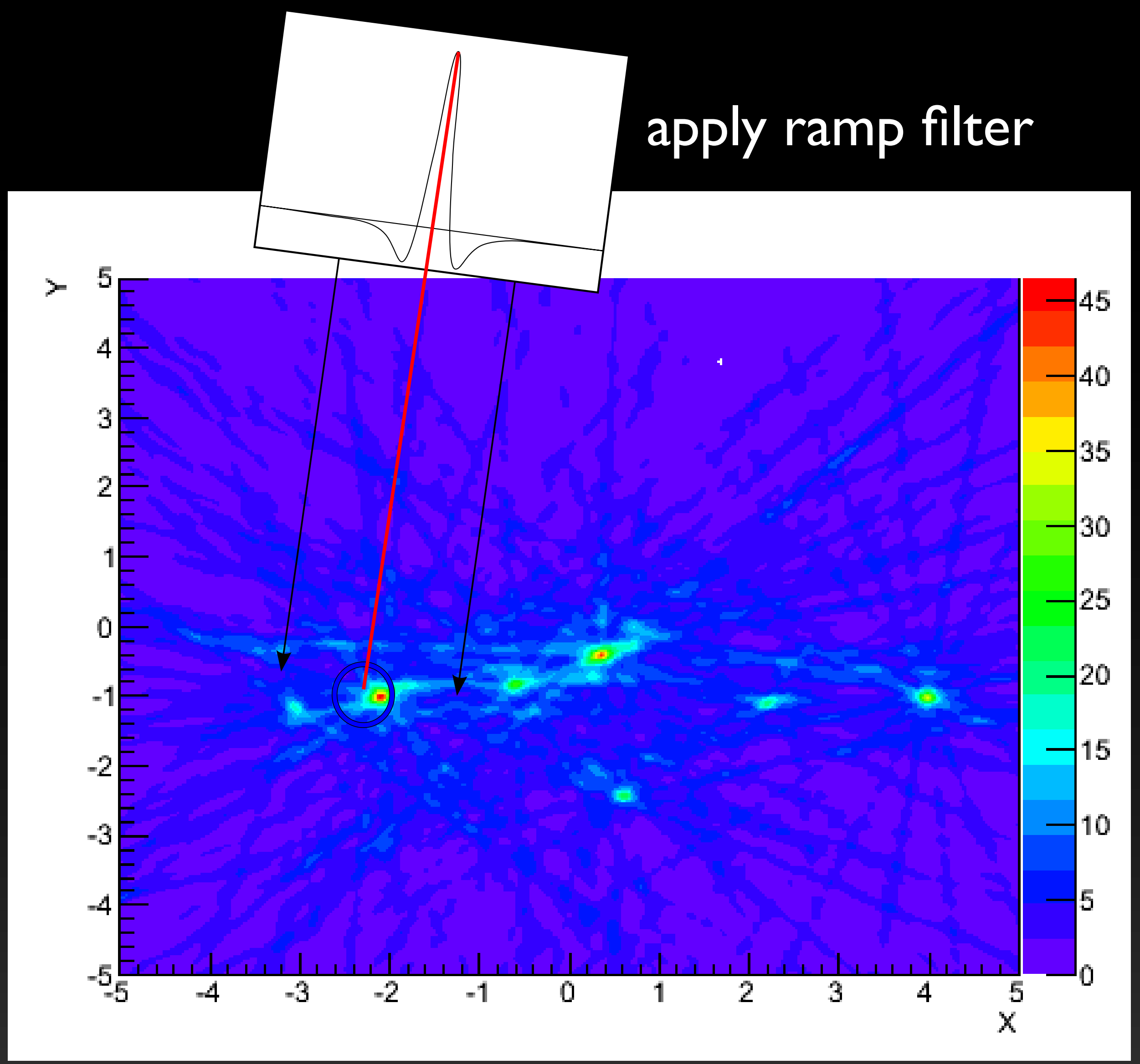
apply ramp filter



- steps for vertex finder:
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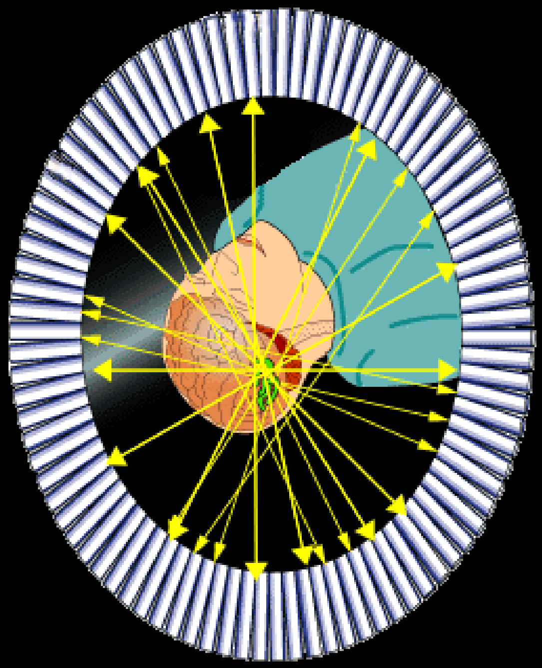
Medical Imaging inspired Vertexing



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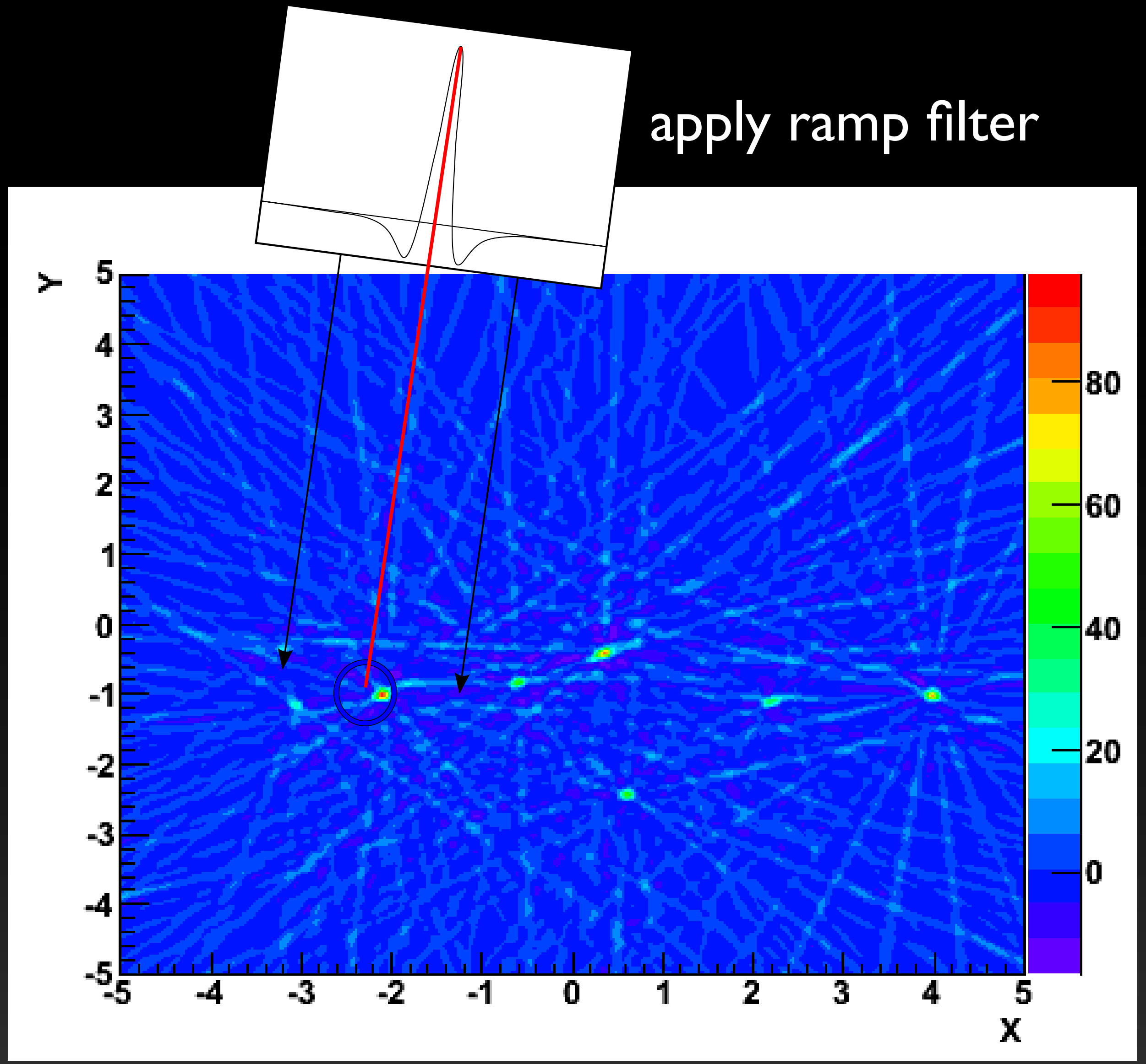
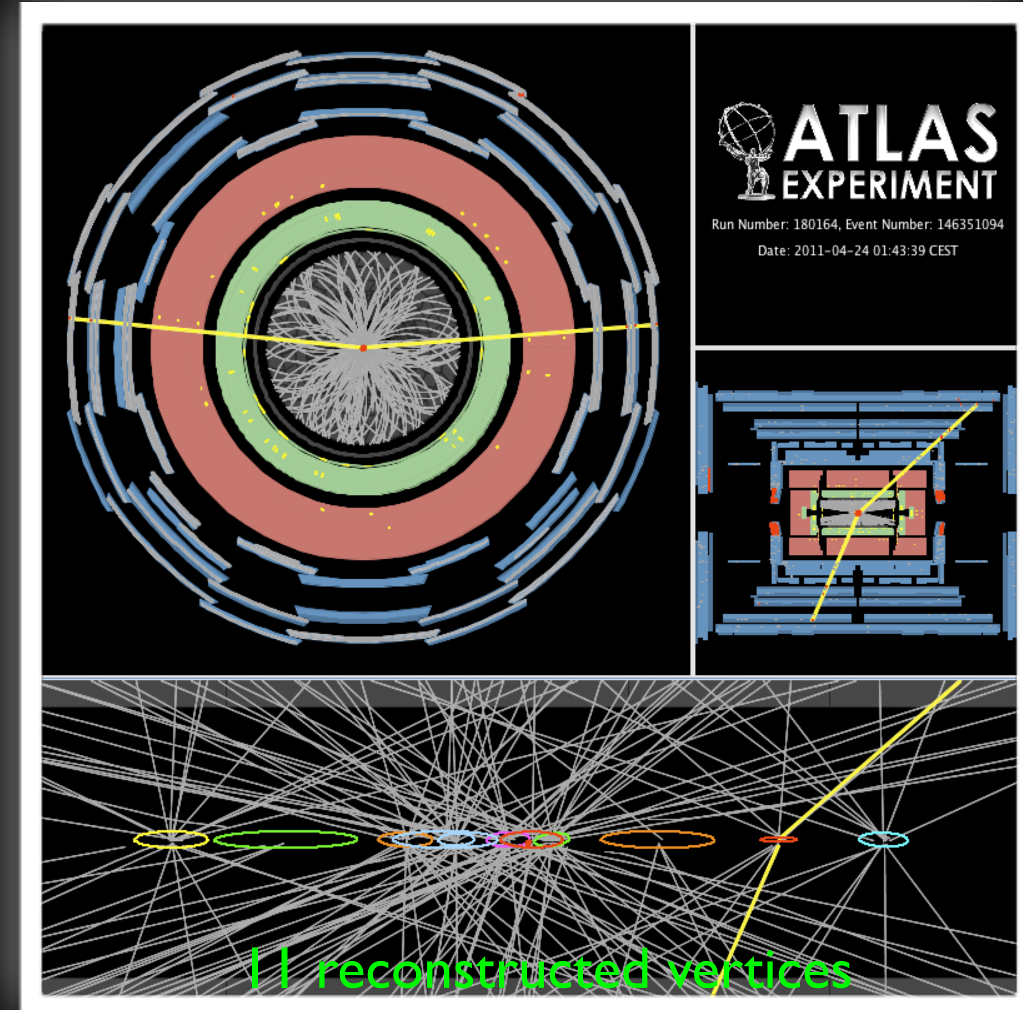


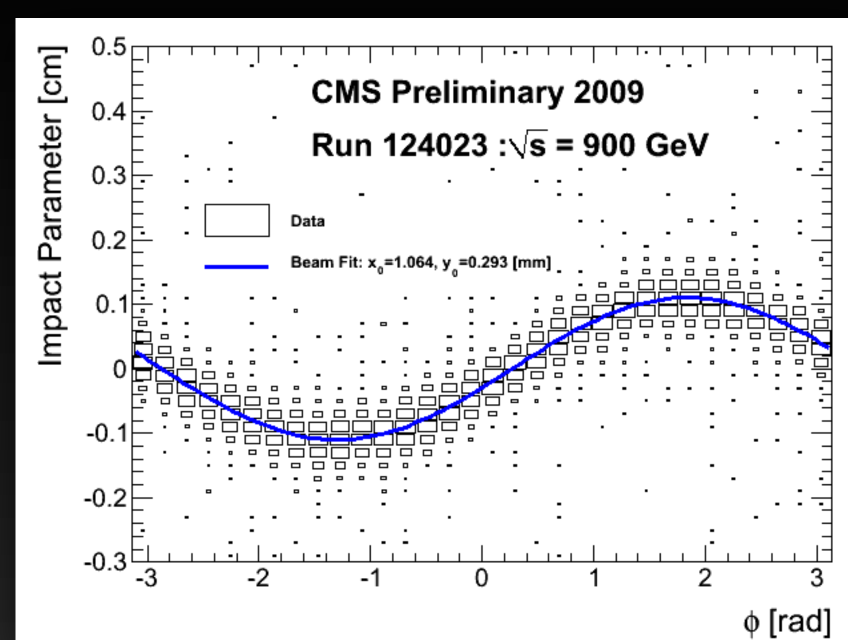
- steps for vertex finder:
  - ➡ create 3D track density maps
  - ➡ Fourier transform Gaussian tubes
  - ➡ apply (ramp) k-filter
  - ➡ back transform image
  - ➡ find vertex candidates as local maxima
  - ➡ fits (like ZVTOP)

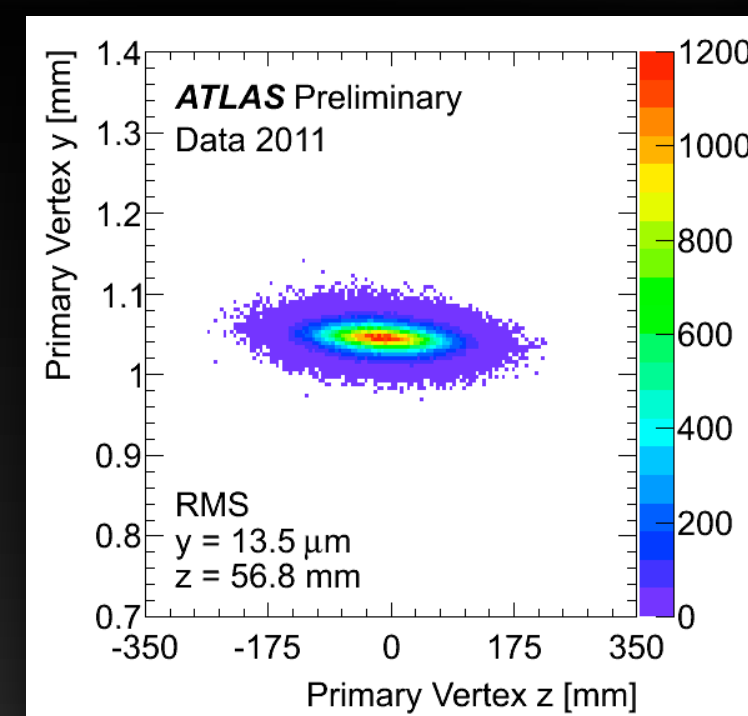
- sharper image, but more noise

Vertexing Applications

- primary vertex finding
  - ➡ reconstruct primary and pileup vertices
  - ➡ ATLAS (and CMS) use an iterative vertex finder and an adaptive fitter
- beam spot routinely determined
  - ➡ averaged over short periods of time
  - ➡ input to primary vertex reconstruction as a constraint
- many applications
  - ➡ primary vertex counting (luminosity)
  - ➡ jet energy scale correction for in time pileup
  - ➡ ...



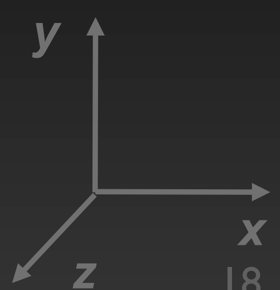




b-Jet Tagging

- several different techniques being explored to tag b-(and c-) jets
  - ➡ explore b-(c-) hadron fragmentation, lifetimes, mass and decay properties

- 3 categories:
  - ➡ **soft lepton** tagging
    - explore semileptonic b- and c-decays (BR~10%)
    - tagging is done using p_T of lepton to jet axis
  - ➡ **impact parameter** tagging
    - tagging is done using IP significance w.r.t. PV
    - sign impact parameter (IP) w.r.t. jet axis
    - done in $R\phi$ (2D) or in $R\phi$+Rz (3D)
  - ➡ **secondary vertex** (SV) tagging
    - reconstruct b-(c-)decay vertex
    - use decay length significance
    - additional vertex information: mass, multiplicity, total momentum



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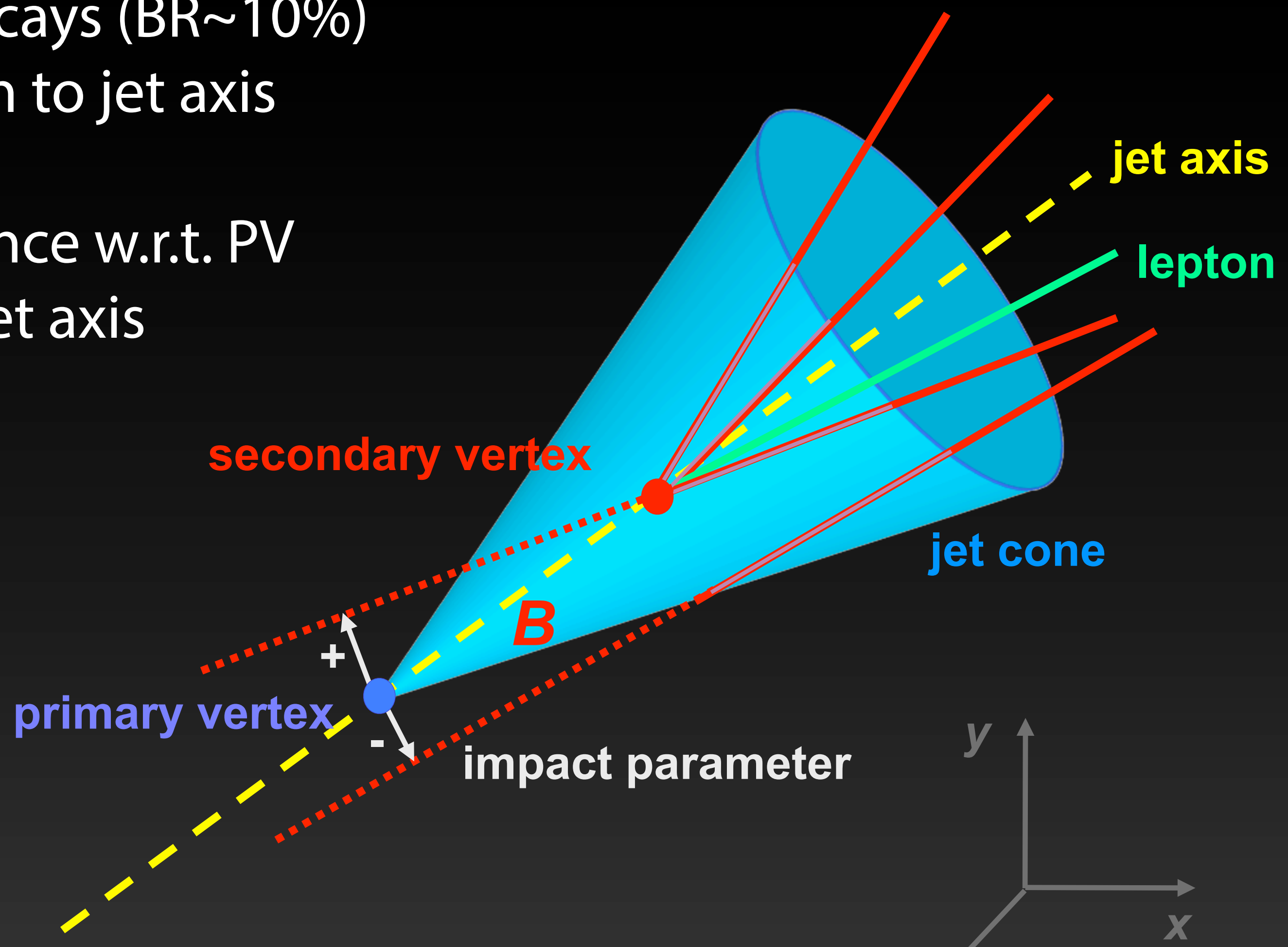
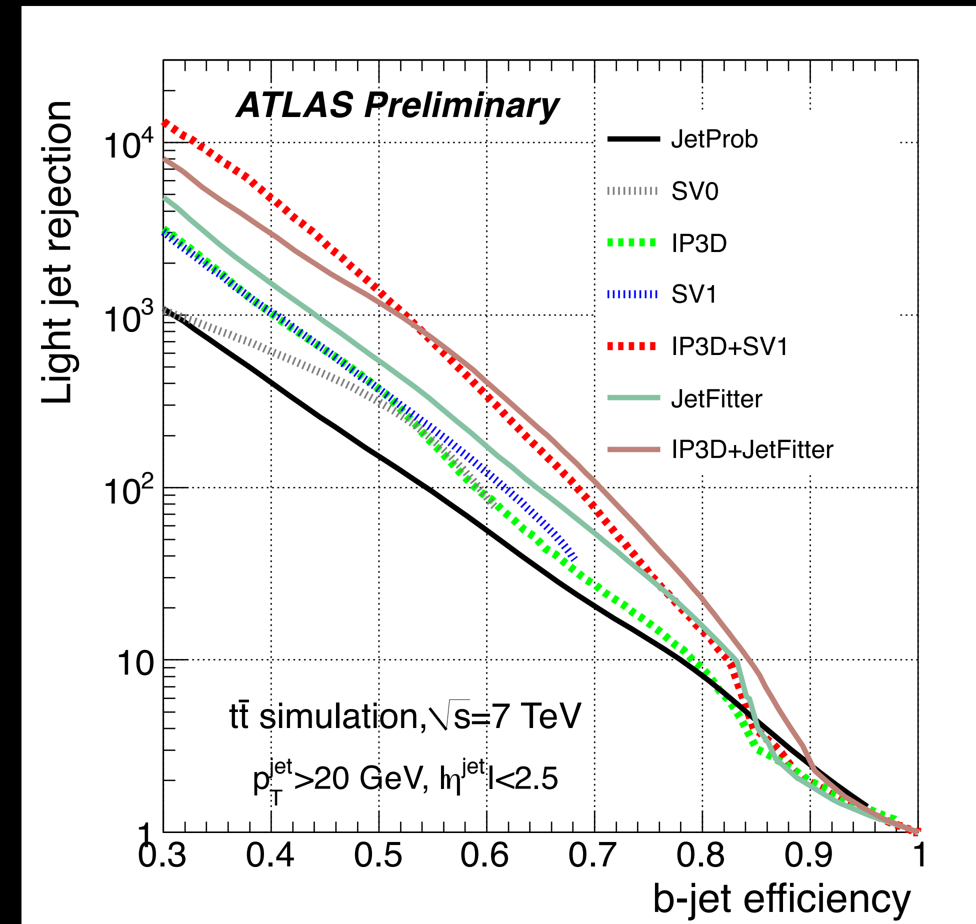
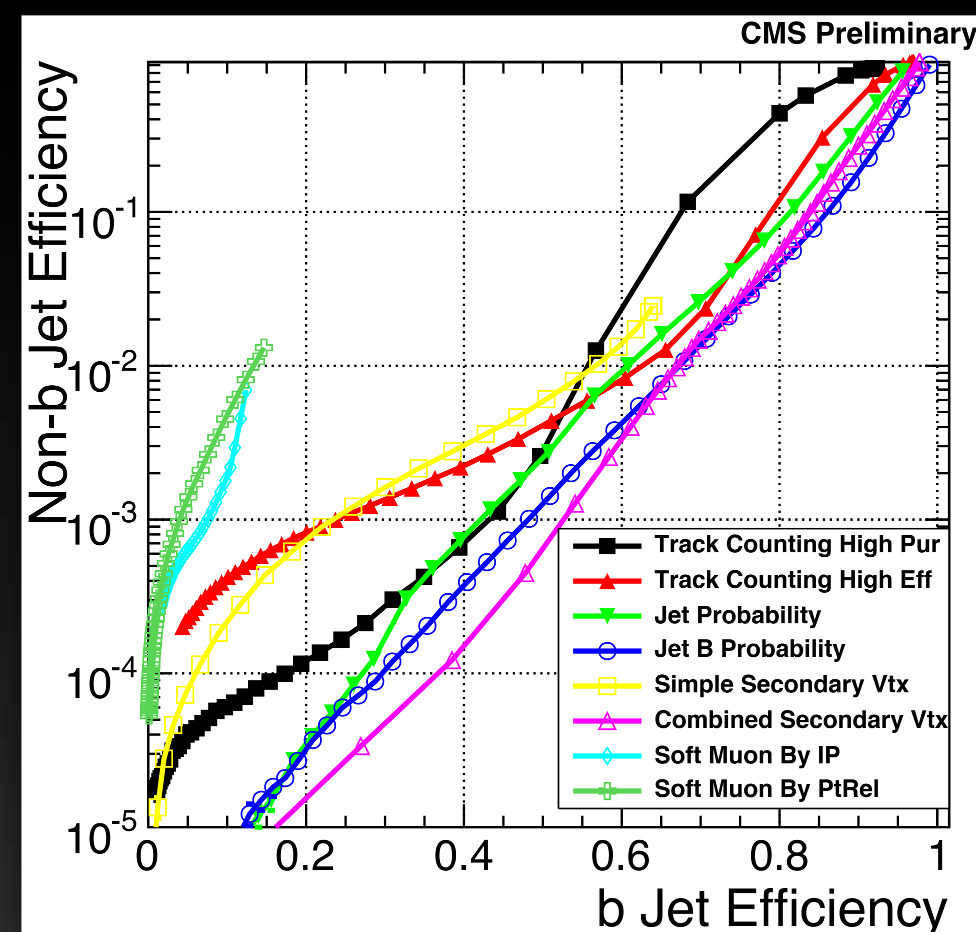
    - explore semileptonic b- and c-decays (BR~10%)
    - tagging is done using p_T of lepton to jet axis
  - ➡ **impact parameter** tagging
    - tagging is done using IP significance w.r.t. PV
    - sign impact parameter (IP) w.r.t. jet axis
    - done in Rϕ (2D) or in Rϕ+Rz (3D)
  - ➡ **secondary vertex** (SV) tagging
    - reconstruct b-(c-)decay vertex
    - use decay length significance
    - additional vertex information: mass, multiplicity, total momentum



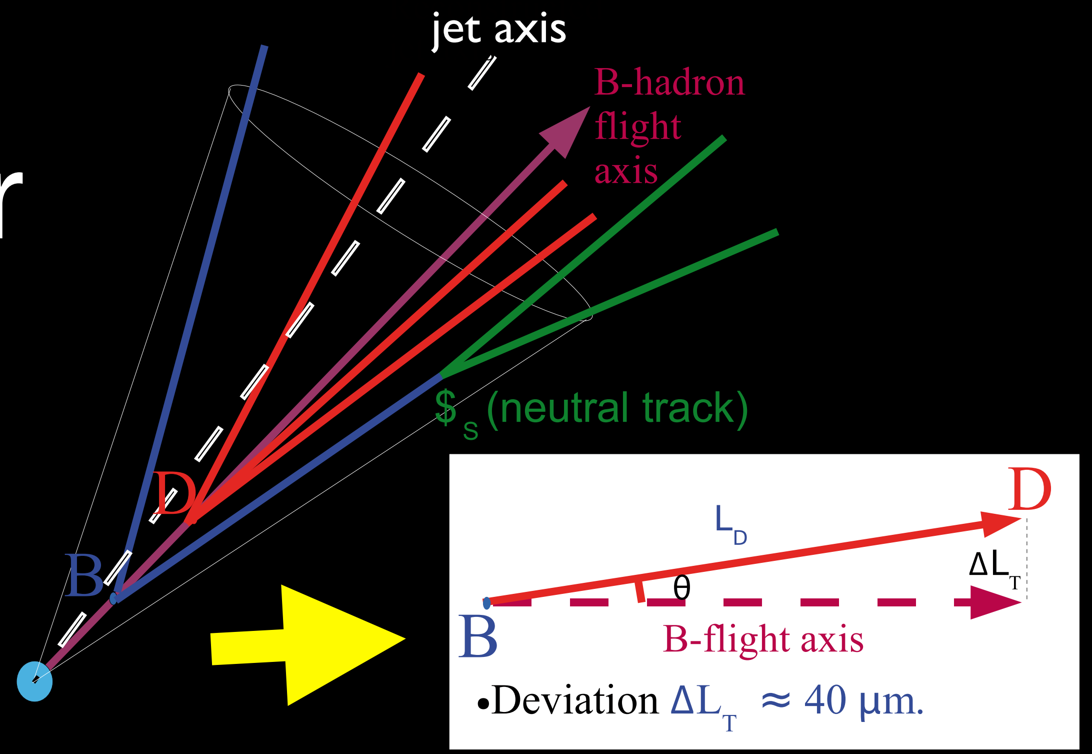
b-Jet Tagging

- ‘simple’ tagging techniques
  - ➡ **soft lepton** tagger
  - ➡ **track counting**
    - count number of tracks significant IP offsets
  - ➡ **jet probability**
    - construct probability that IP significance of all tracks in jet is compatible with PV
  - ➡ **secondary vertex** (SV) tagger
    - decay length significance
- more elaborate combined taggers
  - ➡ construct IP based likelihood using b/c/light templates (**IP2D** and **IP3D**)
  - ➡ combined likelihood taggers using IP and secondary vertex information (**IP3D+SV0**)
  - ➡ use multi-variant techniques to classify jets (baseline today)
- similar set of algorithms used by experiments

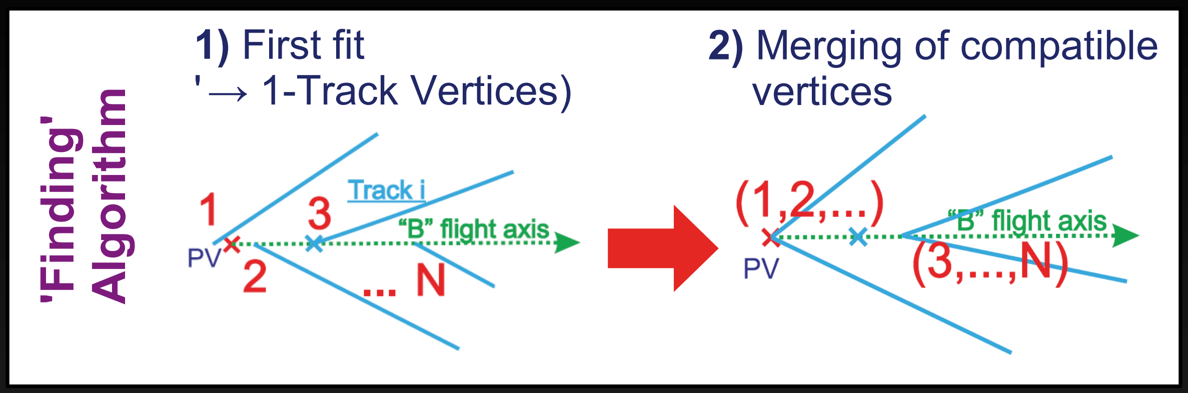


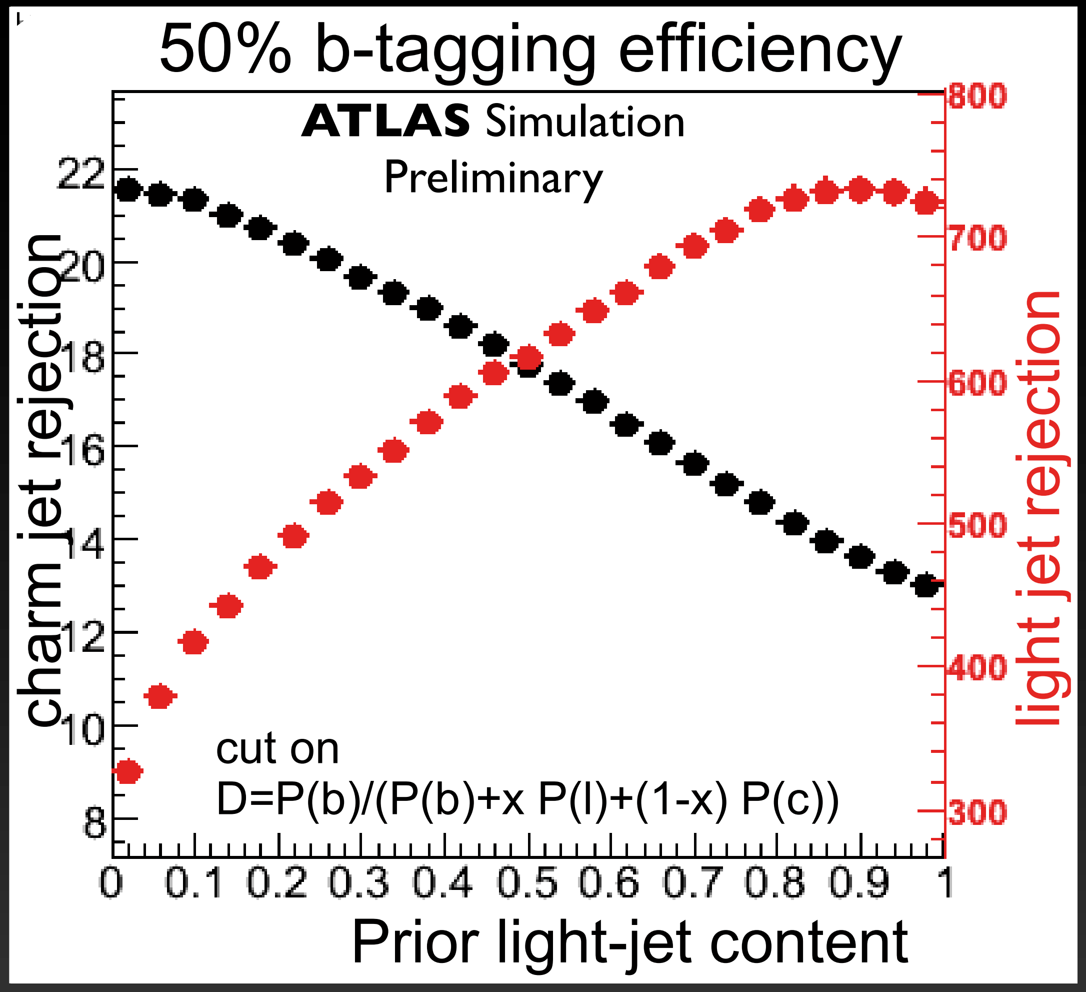


Jet-Fitter as a b-Tagger



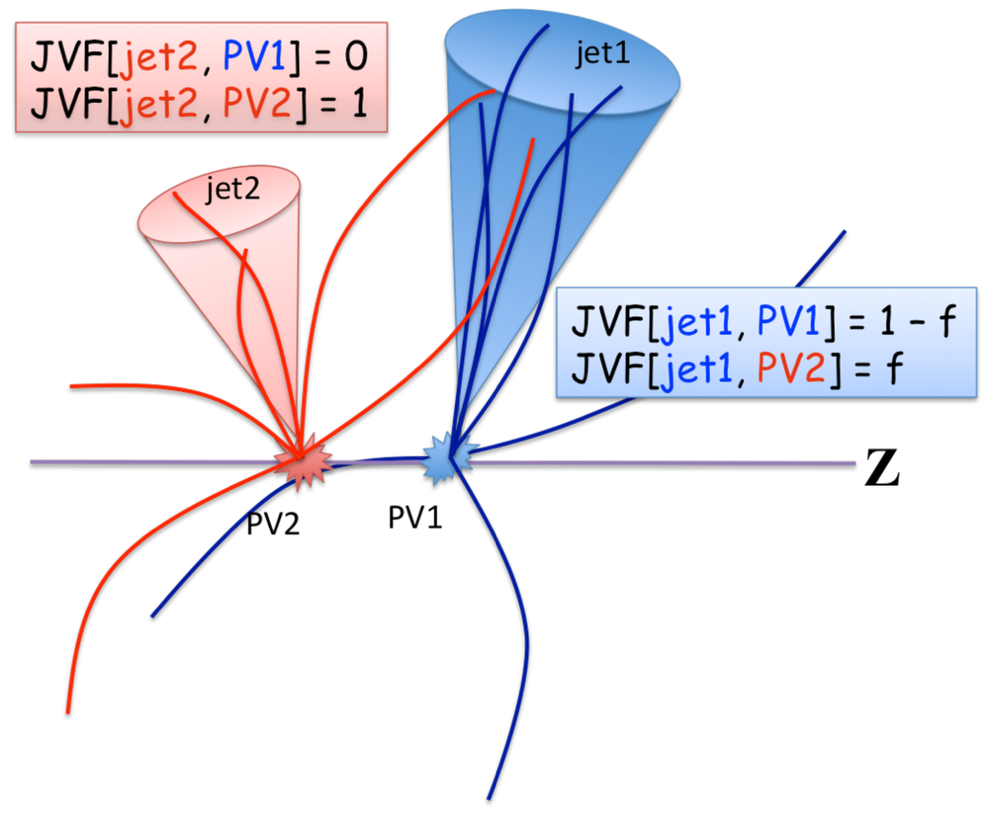
- conventional vertex tagger
  - ➡ fits all displaced tracks into a common geometrical vertex
- Jet-Fitter
  - ➡ b-/c-hadron vertices and primary vertex approximately on the same axis
  - ➡ fit of 1..N vertices along B-hadron axis
  - ➡ mathematical extension of conventional Kalman Filter vertex fitter





- up to 40% better light rejection
  - ➡ much improved control of charm rejection
  - ➡ best b-tagger in ATLAS

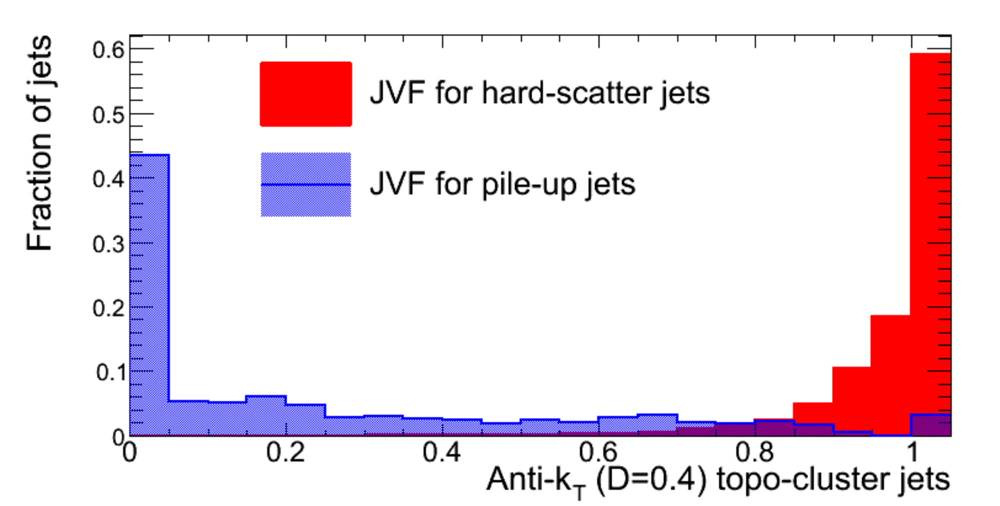
Jet-Vertex-Fraction

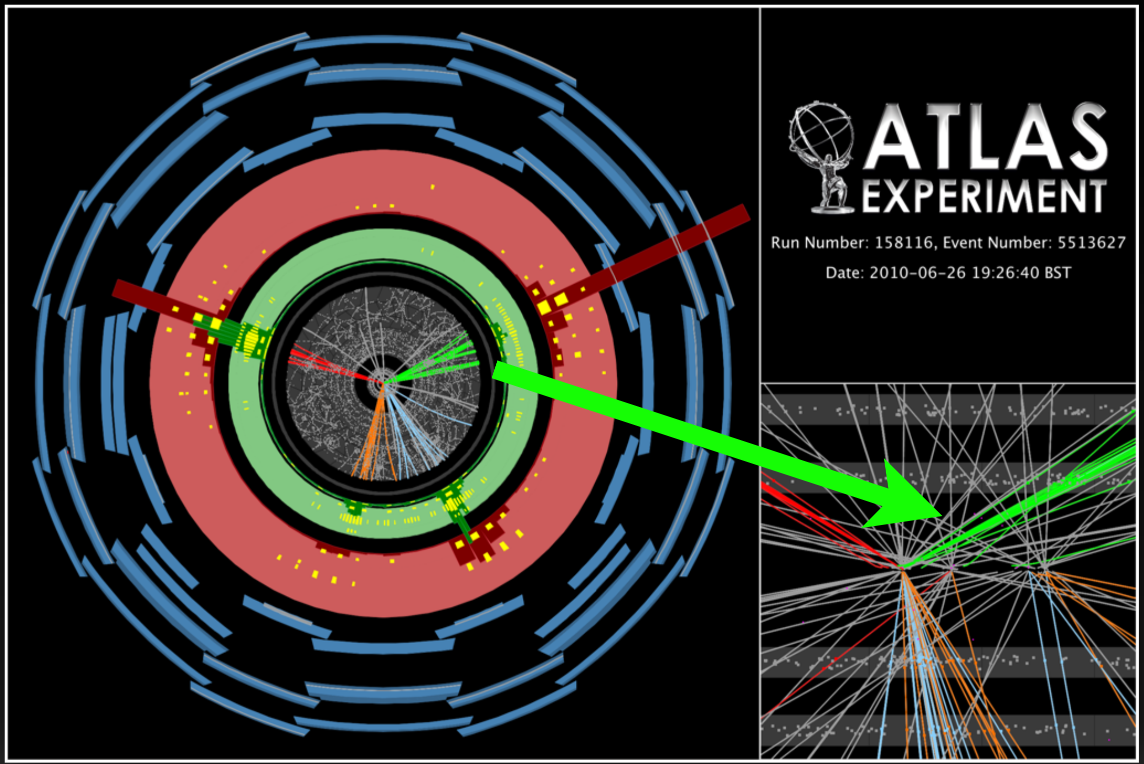


- developed at D0
  - ➡ separate jets from signal and pileup events
  - ➡ defined fraction of pT of tracks in jets associated to primary vertex:

$$JVF(\text{jet}_i, \text{vtx}_j) = \frac{\sum_k p_T(\text{trk}_k^{\text{jet}_i}, \text{vtx}_j)}{\sum_n \sum_l p_T(\text{trk}_l^{\text{jet}_i}, \text{vtx}_n)}$$

  - ➡ good separation in D0 and at LHC at low pileup





- LHC interaction region is a factor ~6 smaller than at Tevatron
  - ➡ more confusion at LHC design luminosity

Let's Summarize...

- discussed vertex fitting and finding techniques
- b-tagging and other examples for vertexing applications
- next is to discuss lessons from early data taking to conclude lecture series